

# **Broadened-Specular Reflection and Linear Subspaces for Object Recognition**

## **Background of the Invention**

### **Field of Invention**

[0001] The present invention relates generally to machine vision, and more particularly, to a method of modeling the broadened-specular reflection of light to allow a machine to recognize the image of an object illuminated by varying lighting conditions.

### **Description of Related Art**

[0002] It is clear that changes in lighting can have a large effect on how an object, such as a person, looks. Understanding this is essential to building effective object recognition systems.

[0003] U.S. Patent Application Serial Number 09/705,507 (Attorney Docket No. NECI 1093, 13919), filed 3 November 2000, entitled "Lambertian Reflectance and Linear Subspaces", the disclosure of which is hereby incorporated by reference, considered the relationship between the

function that describes the lighting intensity, and the reflectance function that describes how much light an object reflects as a function of its surface normal, under a given lighting condition. Representing these functions as spherical harmonics, it was shown that for Lambertian reflectance the mapping from lighting to reflectance is a convolution with a nearly perfect low-pass filter. High-frequency components of the lighting hardly affect the reflectance function. Therefore, nearly all Lambertian reflectance functions could be modeled as some linear combination of nine spherical harmonic components.

**[0004]** These low-dimensional approximations have two key advantages. First, they are described with few parameters, thus optimization is simpler. Moreover, the images produced under low-frequency lighting are inherently immune to small changes in orientation. A small rotation in the surface normal is equivalent to a small rotation in the light. However, low-frequency light does not appreciably change with a small rotation. The analysis shows this, in that an image produced by an  $n^{\text{th}}$  order harmonic light component is computed by taking an  $n^{\text{th}}$  degree polynomial of the components of the surface normal.



Detecting Specular Reflection Using Lambertian Constraints,  
*International Conference on Computer Vision*, pp. 297-302  
(1988), treat specular pixels as statistical outliers that  
should be discarded. However, specularities due to low-  
frequency lighting can be spread out throughout the entire  
image. Since these specularities affect every pixel in the  
image, their effect must be better understood; they cannot  
simply be discarded.

[0008] Some previous works have considered specular effects  
also as convolutions. Basri, et al., (August 2000)  
considered a version of the Phong model in which the  
specular lobe is a rotationally symmetric shape centered  
about a peak that is a 180-degree rotation of the direction  
of the incoming light about the surface normal. In such a  
model, specular reflection doubles the frequency of the  
incoming light, and then acts as a convolution. However,  
this model is heuristic, and not physically based. It does  
not extend to realistic models.

[0009] Ramamoorthi, et al., On the Relationship between  
Radiance and Irradiance: Determining the Illumination From  
Images of a Convex Lambertian Object, to appear in *Journal  
of the Optical Society of America A*, considered the effects  
of arbitrary bi-directional reflectance distribution

functions (BRDFs) on the light field. They were able to develop algorithms for recovering the BRDF and/or the lighting given a measured light field and an object with a known 3D structure. Though interesting, these results are not generally applicable to machine vision because, without special sensing devices, the light field is not available. An image of an object is produced by the reflectance function that describes how the surface normals reflect light in one fixed direction. Therefore, an image corresponds to a 2D slice through the light field. For an arbitrary BRDF, the 2D reflectance function for a fixed viewpoint no longer results from a convolution of the lighting.

### **Brief Summary of the Invention**

**[0010]** The goal of the present invention is to allow a machine, such as a computer, to match an input image of an object, such as a human face, to an object model stored in a database associated with the machine, and to perform this matching with a high degree of accuracy.

**[0011]** The steps of the recognition method of the present invention include providing a database of three-dimensional

09967206-092801  
T03260-90279660

object models, providing an input image, positioning each three dimensional model relative to the input image, determining an optimal rendered image of the model that is most similar to the input image, computing a measure of similarity between the input image and the optimal rendered image of each model, and selecting the object whose optimal rendered image is most similar to the input image. The step of determining an optimal rendered image includes deriving a reflectance function for each object that includes the effects of both broadened-specular and diffuse reflection, and optimizing that reflectance function to most closely match the input image.

**[0012]** The method of the present invention includes rendering the images produced by an object model when illuminated only by each of a plurality of spherical harmonic components of light. The summation of these components defines the reflectance function for the object model. The derived reflectance function takes into account the intensity of light components incident on the object model, the intensity of light diffusely reflected from the model, and the intensity of light specularly reflected from the model, the last component being dependent on the angle

between the direction of the observer and the direction of specular reflection.

[0013] To overcome the limitations of dealing with specular reflection, a two-pronged strategy is used. The light in low-frequency and higher-frequency components are analyzed separately. The low-frequency components are analyzed using the method of the present invention, and the results of this analysis are evaluated to determine if they are sufficient to match an optimal rendered image of the object model to the input image.

### **Detailed Description of the Invention**

[0014] Although detailed and/or microscopic models of reflection exist, these apply principally to homogeneous, rigid, material surfaces. For reflection from inhomogeneous, flexible surfaces such as faces, where most of the features of interest are configurational, a more robust model that captures the most important aspects of the scattered light is more appropriate. Thus the Lambertian model, which simply scatters incident radiation equally into the hemisphere above each surface element, is usually chosen.

108250 0924960

[0015] However, due to broadened-specular reflection, in viewing a person's face, regions of unusually high reflectivity are apparent. The intensity of light reflect from these regions is sensitive to the viewing direction. As this aspect of the illuminated face cannot be captured by Lambertian scattering, even in principle, the Lambertian model is supplemented with the Phong model. The latter scatters the incoming light preferentially into directions distributed about the direction of purely specular reflection. In addition, the distribution of scattered directions is readily analyzable in the spherical-harmonic representation, in a manner similar to that of the Lambertian model. Appropriately weighted, this additional and more selective scattering should adequately model the human face for most lighting conditions of interest.

[0016] Though an enhancement of the Phong model of reflection was chosen, the method of the present invention need not be based upon the Phong model. Any model that depends only on the cosine of the angle between the reflected light and the direction of perfect specular reflection can be treated in the same manner.

[0017] While not completely dominant, low-frequency lighting will have a considerable effect on the



reflectances due to the broadened-specular component of an object. This is because many situations contain significant low-frequency lighting, such as outdoors on a cloudy day, and many indoor settings.

**[0018]** The method of the present invention shows how to analyze the reflectance function that is produced by specular reflection for more physically realistic BRDFs. This is accomplished by adding more realistic terms to the Phong model. These terms modulate the Phong model by the amount of light that is actually striking a surface patch. The new model allows the development of techniques that will be relevant for more physically realistic models, while avoiding some of the complexity of those models.

**[0019]** The physically realistic model leads to the problem of taking integrals in three terms: the lighting intensity, which is a function of its incoming direction; the specular term, which is a function of the angle between the specular peak and the viewing direction; and the amount of light striking the surface, which is a function of the angle between the incoming light and the surface normal. After expanding these terms in spherical harmonics, this can be rewritten as the integral of the product of three harmonics. This problem has been solved in other areas of

research, for example atomic spectroscopy. The solution may be thought of as a generalization of the convolution theorem to this case.

**[0020]** The results can be used to determine the set of images that an object produces when lighting consists of a single harmonic component. Therefore, one can analytically characterize the images an object produces, and determine how the effect of low- and high-frequency lighting on an image varies with the parameters of the Phong model.

**[0021]** Conceptually, the method of deriving the reflectance function of an object model according present invention involves transforming the function of light intensity incident upon an object into its low-dimensional spherical harmonic components. In the present sense, low-dimensional is used to mean the 9D subspace formed by the summation of the 0<sup>th</sup>, 1<sup>st</sup>, and 2<sup>nd</sup> order spherical harmonic components. Each component is applied to the surface normal at each point on the object. Using the reflectance model described herein, the reflectance function is determined, and described in terms of the same spherical harmonic components. These harmonic components can then be summed to produce a rendered image, a linear representation of the image of the object model. As a practical matter, however,

the machine can utilize the harmonic components and optimize their weightings to determine if the image received is a match to some known object in its database.

[0022] The technique of analyzing the reflectance function in terms of spherical harmonic components follows.

[0023] Given  $L(\hat{\mathbf{n}}_i)$ , the intensity of light incident in the  $-\hat{\mathbf{n}}_i$  direction on a perfectly reflecting, smooth surface with outward pointing normal  $\hat{\mathbf{n}}$ , the reflected light  $r_s(\hat{\mathbf{n}}_e, \hat{\mathbf{n}})$  in direction  $\hat{\mathbf{n}}_e$  can be expressed as

$$r_s(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \int d\hat{\mathbf{n}}_i L(\hat{\mathbf{n}}_i) \delta(\hat{\mathbf{n}}_e - \hat{\mathbf{n}}_r)$$

where  $\hat{\mathbf{n}}_r = 2\hat{\mathbf{n}}(\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}) - \hat{\mathbf{n}}_i$  is the direction of specular reflection corresponding to incoming light along  $-\hat{\mathbf{n}}_i$ . (Note  $\hat{\mathbf{n}}_r$  lies in the  $(\hat{\mathbf{n}}, \hat{\mathbf{n}}_i)$  plane,  $\hat{\mathbf{n}}_r \cdot \hat{\mathbf{n}}_r = 1$ , and  $\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_r = \hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i$ . Here  $u(x)$  is the unit step function:  $u(x) = 0, x < 0, u(x) = 1, x > 0$ .) The prefactor  $u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}})$  ensures that the orientation  $\hat{\mathbf{n}}$  of the surface permits the reflection of light in direction  $\hat{\mathbf{n}}_e$ . As  $(\hat{\mathbf{n}}_e - \hat{\mathbf{n}}_r) = \mathbf{R}(\hat{\mathbf{n}}'_e - \hat{\mathbf{n}}_i)$ , where  $\hat{\mathbf{n}}'_e = 2\hat{\mathbf{n}}(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) - \hat{\mathbf{n}}_e$  and  $\mathbf{R}$  is a rotation of  $\pi$  about  $\hat{\mathbf{n}}$ ,  $\delta(\hat{\mathbf{n}}_e - \hat{\mathbf{n}}_r) = \delta(\hat{\mathbf{n}}'_e - \hat{\mathbf{n}}_i)$ , and since

$$\delta(\hat{\mathbf{n}}_1 - \hat{\mathbf{n}}_2) = \sum_{lm} Y_{lm}(\hat{\mathbf{n}}_1) Y_{lm}^*(\hat{\mathbf{n}}_2),$$

where  $Y_{lm}(\hat{\mathbf{n}})$  is the  $lm$  spherical harmonic evaluated in direction  $\hat{\mathbf{n}}$ ,  $r_s(\hat{\mathbf{n}}_e, \hat{\mathbf{n}})$  can be expressed compactly as

$$r_s(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \sum_{lm} L_{lm} Y_{lm}(2\hat{\mathbf{n}}(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) - \hat{\mathbf{n}}_e),$$

where the  $L_{lm} = \int d\hat{\mathbf{n}}_i L(\hat{\mathbf{n}}_i) Y_{lm}^*(\hat{\mathbf{n}}_i)$  are the components (strengths) of the incident intensity expressed in spherical harmonics.

Choosing  $\hat{\mathbf{n}}_e$  to be the  $\hat{\mathbf{z}}$  axis, and noting that  $\hat{\mathbf{n}}'_e = \mathbf{R}\hat{\mathbf{n}}_e$  is now simply the rotation of the  $\hat{\mathbf{z}}$  axis about  $\hat{\mathbf{n}}$  by  $\pi$ ,  $Y_{lm}(\mathbf{R}\hat{\mathbf{z}})$  becomes  $Y_{lm}(\beta, \alpha)$ , where  $\alpha$  and  $\beta$  are the first two Euler angles of  $\mathbf{R}$ . This is discussed in more detail below in connection with the Phong model. Note especially that as the polar angle of the surface normal  $\hat{\mathbf{n}}$  varies from 0 to  $\pi/2$ , the polar angle  $\beta$  varies from 0 to  $\pi$ , as expected for perfect reflection.

[0024] For comparison, one can derive the reflected light from a Lambertian surface  $r_L(\hat{\mathbf{n}}_e, \hat{\mathbf{n}})$  in a similar manner.

$$r_L(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \int d\hat{\mathbf{n}}_i L(\hat{\mathbf{n}}_i) (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i \vee 0)$$

where  $(A \vee B) = \max(A, B)$ . A Lambertian surface scatters the incident light equally in all directions  $\hat{\mathbf{n}}_e$  such that

$(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) > 0$ . The  $(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i)$  factor weights the incident flux density in the  $-\hat{\mathbf{n}}_i$  direction. Since

$$L(\hat{\mathbf{n}}_i) = \sum_{lm} L_{lm} Y_{lm}(\hat{\mathbf{n}}_i)$$

and  $(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i \vee 0) = \sum_{lm} (4\pi / (2l+1))^{1/2} k_l Y_{lm}^*(\hat{\mathbf{n}}_i) Y_{lm}(\hat{\mathbf{n}})$  one can obtain the Basri-Jacobs (B-J) Expression:

$$r_L(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \sum_{lm} L_{lm} (4\pi / (2l+1))^{1/2} k_l Y_{lm}(\hat{\mathbf{n}})$$

The  $k_l$  coefficients calculated by B-J fall off as  $l^{-2}$ , and vanish for  $l \geq 3$ , if  $l$  is odd. This, coupled with constraints on the size of the  $L_{lm}$  (as  $L(\hat{\mathbf{n}}_i) \geq 0$ ) enabled them to truncate the sum over  $l$  at  $l=2$ .

[0025] Such a truncation is clearly not possible for specular reflection; however, apart from mirrors, such reflection is seldom present. One model which retains much of the directionality of specular reflection, but introduces a controlled amount of lateral diffusion is the Phong Model, in terms of which one can write

$$r_p(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \int d\hat{\mathbf{n}}_i L(\hat{\mathbf{n}}_i) (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i \vee 0) (\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}_r \vee 0)^p$$

where as above  $\hat{\mathbf{n}}_r = 2\hat{\mathbf{n}}(\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}) - \hat{\mathbf{n}}_i$ . Here the last term distributes the incident light into cones of decreasing intensity about the nominal direction of (specular) reflection  $\hat{\mathbf{n}}_r$ : the larger the exponent  $p$ , the sharper the distribution. This expression reduces to  $r_l$  in the limit that  $p \rightarrow 0$ . As such it may over-emphasize the diffuseness of the scattering. Also, Phong scattering is intended to augment rather than replace Lambertian scattering. An expression that reduces to  $r_s$  in the limit  $p \rightarrow \infty$  is:

$$r'_p(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \int d\hat{\mathbf{n}}_i L(\hat{\mathbf{n}}_i) u(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i) (\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}_r \vee 0)^p$$

which may overemphasize the specularity of the scattering. Fortunately the analysis is valid for both: one need only substitute a  $k_l^{(0)}$  for  $k_l$  to obtain  $r'_p$  from  $r_p$ .

[0026] In the above manner one can write

$$\begin{aligned} L(\hat{\mathbf{n}}_i) &= \sum_{lm} L_{lm} Y_{lm}(\hat{\mathbf{n}}_i) \\ (\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i \vee 0) &= \sum_{l_1 m_1} (4\pi / (2l_1 + 1))^{1/2} k_{l_1} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1}^*(\hat{\mathbf{n}}_i) \\ u(\hat{\mathbf{n}} \cdot \hat{\mathbf{n}}_i) &= \sum_{l_1 m_1} (4\pi / (2l_1 + 1))^{1/2} k_{l_1}^{(0)} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1}^*(\hat{\mathbf{n}}_i) \\ (\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}_r \vee 0)^p &= \sum_{l_2 m_2} (4\pi / (2l_2 + 1))^{1/2} k_{l_2}^{(p)} Y_{l_2 m_2}(\hat{\mathbf{n}}_e) Y_{l_2 m_2}^*(\hat{\mathbf{n}}_r) \end{aligned}$$

where  $k_l$  is calculated in B-J Appendix A, and  $k_l^{(p)}$  in B-J Appendix C, where  $\alpha$  is used in place of  $p$ .  $k_l^{(0)}$  can be

obtained analogously, and is given below, along with  $k_l$ ;  $k_l^{(p)}$

is given below as well. Combining one can obtain

$$r_p(\hat{\mathbf{n}}_e, \hat{\mathbf{n}}) = u(\hat{\mathbf{n}}_e \cdot \hat{\mathbf{n}}) \sum_{lm} L_{lm} \cdot \sum_{l_1 m_1} (4\pi / (2l_1 + 1))^{1/2} k_{l_1} Y_{l_1 m_1}(\hat{\mathbf{n}}) \cdot \sum_{l_2 m_2} (4\pi / (2l_2 + 1))^{1/2} k_{l_2}^{(p)} Y_{l_2 m_2}(\hat{\mathbf{n}}_e) \cdot \int d\hat{\mathbf{n}}_i Y_{lm}(\hat{\mathbf{n}}_i) Y_{l_1 m_1}^*(\hat{\mathbf{n}}_i) Y_{l_2 m_2}^*(\hat{\mathbf{n}}_i)$$

where the remaining integral over incident direction  $(\hat{\mathbf{n}}_i)$  is independent of  $(\hat{\mathbf{n}}_e)$ . For the purposes of the above

$$k_l = \begin{cases} \pi^{1/2} / 2 & l = 0 \\ (\pi/3)^{1/2} & l = 1 \\ \frac{(-1)^{l+1/2} ((2l+1)\pi)^{1/2} (l-2)!}{2^l (l/2-1)! (l/2+1)!} & l \geq 2, \text{ even} \\ 0 & l \geq 2, \text{ odd} \end{cases}$$

$$k_l^{(0)} = \begin{cases} \pi^{1/2} & l = 0 \\ \frac{(-1)^{(l+1)/2} ((2l+1)\pi)^{1/2} (l-1)!}{2^l ((l-1)/2)! ((l+1)/2)!} & l > 0, \text{ odd} \\ 0 & l > 0, \text{ even} \end{cases}$$

and

$$k_l^{(p)} = \begin{cases} 0 & p < l, (l-p) \text{ even} \\ \frac{((2l+1)\pi)^{1/2} (-1)^{(l+p+1)/2} p!(l-p-1)!}{2^l ((l-p-1)/2)! ((l+p+1)/2)!} & p < l, (l-p) \text{ odd} \\ \frac{((2p+1)\pi)^{1/2} 2^{p-1} p!(p-1)!}{(2p+1)(2p-1)!} & p = l \\ \frac{((2l+1)\pi)^{1/2} 2^{l+1} p!((p+l)/2)!}{(p-l)(p+l+1)!((p-l)/2-1)!} & p > l, (p-l) \text{ even} \\ \frac{((2l+1)\pi)^{1/2} p!((p-l-1)/2)!}{2^{l+1} (p-l)!((p+l+1)/2)!} & p > l, (p-l) \text{ odd} \end{cases}$$

[0027] Without loss of generality one can take the direction of observation  $-\hat{\mathbf{n}}_e$  to be the  $-\hat{\mathbf{z}}$  axis of our coordinate system, as is commonly chosen. This is also the system in which the lighting is specified. As  $Y_{lm}(\hat{\mathbf{z}})$  is zero unless  $m=0$ ,  $Y_{l0}(\hat{\mathbf{n}}_r)$  must be expressed in terms of  $Y_{lm}(\hat{\mathbf{n}}_i)$  in order to carry out the final integral over  $\hat{\mathbf{n}}_i$ .

[0028] Proceeding as follows:  $\hat{\mathbf{n}}_r = 2\hat{\mathbf{n}}(\hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}) - \hat{\mathbf{n}} = \mathbf{R}\hat{\mathbf{n}}_i$  represents a rotation  $\mathbf{R}$  of  $\hat{\mathbf{n}}_i$  around by  $\hat{\mathbf{n}}$  by  $\pi$  (or by  $-\pi$ ). Though it may intuitively seem more natural to view the relation between  $\hat{\mathbf{n}}_i$  and  $\hat{\mathbf{n}}_r$  as a rotation in the plane of incidence defined by  $\hat{\mathbf{n}}_i$  and  $\hat{\mathbf{n}}$  about an axis through  $\hat{\mathbf{n}}_i \times \hat{\mathbf{n}}$ , using  $\mathbf{R}$  instead effects numerous simplifications. From the relation between  $\hat{\mathbf{n}}_i$  and  $\hat{\mathbf{n}}_r$  it is clear that

$$\mathbf{R} = 2\hat{\mathbf{n}}\hat{\mathbf{n}}^T - \mathbf{I}$$



whose representation in terms of Euler angles can be found by first equating  $R_{33}$  entries

$$\cos \beta = R_{33} = 2n_z^2 - 1, \sin \beta = 2|n_z|(1 - n_z^2)^{1/2}$$

and then equating  $R_{31}$  entries

$$R_{31} = 2n_z n_x = \sin \beta \cos \alpha; \quad \cos \alpha = n_z n_x / |n_z|(n_x^2 + n_y^2)^{1/2}$$

[0029] Since  $\mathbf{R}$  is symmetric,  $\cos \gamma = -\cos \alpha$  and  $\sin \gamma = \sin \alpha$ .

Thus  $\gamma = (\pi - \alpha)$ . Also,  $\sin \alpha = n_z n_y / |n_z|(n_x^2 + n_y^2)^{1/2}$ . (Here  $(\alpha, \beta, \gamma)$

are the three Euler angles representing  $\mathbf{R}$ , and  $\hat{\mathbf{n}} = (n_x, n_y, n_z)$ .

Strictly speaking, the Euler angles refer to a rotation of the coordinates axes while the vectors remain fixed.

Hence, one should be concerned with representing  $\mathbf{R}^T$ ;

however, since  $\mathbf{R}$  is symmetric,  $\mathbf{R} = \mathbf{R}^T$ .) Since  $\hat{\mathbf{z}} \cdot \hat{\mathbf{n}} > 0, n_z > 0$ ,

and  $n_z / |n_z| = 1$ .

[0030] Now, to evaluate  $Y_{l0}(\mathbf{R}\hat{\mathbf{n}}_i)$ , the polar angle  $\theta_r$  between

$\hat{\mathbf{n}}_i$  and the rotated  $\hat{\mathbf{z}}$ -axis is needed. However, in the

original system, the rotated  $\hat{\mathbf{z}}$ -axis lies in the

$\hat{\mathbf{n}}_{\beta\alpha} = (\theta = \beta, \phi = \alpha)$  direction, and  $\hat{\mathbf{n}}_i$  is unchanged. Hence

$\cos \theta_r = \hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_{\beta\alpha}$  and, as can be shown

$$Y_{l0}(\hat{\mathbf{n}}_i) = Y_{l0}(\mathbf{R}\hat{\mathbf{n}}_i) = (4\pi / (2l+1))^{1/2} \sum_m Y_{lm}^*(\beta, \alpha) Y_{lm}(\hat{\mathbf{n}}_i)$$

The expression for the reflected light based upon the Phong model then becomes

$$r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}}) = u(\hat{\mathbf{z}} \cdot \hat{\mathbf{n}}) \sum_{lm} L_{lm} \cdot \sum_{l_1 m_1} (4\pi / (2l_1 + 1))^{1/2} k_{l_1} Y_{l_1 m_1}(\hat{\mathbf{n}}) \cdot \sum_{l_2 m_2} (4\pi / (2l_2 + 1))^{1/2} k_{l_2}^{(p)} Y_{l_2 m_2}(\beta, a) \cdot \int d\hat{\mathbf{n}}_i Y_{lm}(\hat{\mathbf{n}}_i) Y_{l_1 m_1}^*(\hat{\mathbf{n}}_i) Y_{l_2 m_2}^*(\hat{\mathbf{n}}_i)$$

It remains to evaluate the latter integral.

[0031] The latter integral can be readily performed if the product of the second pair of spherical harmonics can be written as a sum of spherical harmonics. Such problems were solved in the context of the theory of angular momentum, a key component of present understanding of atomic spectroscopy. The result for this integral is that it is equal to

$$((2l_1 + 1)(2l_2 + 1) / 4\pi(2l + 1))^{1/2} C(l_1 l_2 l; m_1 m_2 m) C(l_1 l_2 l; 000)$$

where the C are Clebsch-Gordon (C-G) coefficients; simple, real numbers, readily calculable, and equal to zero unless  $m = m_1 + m_2, |m_1| \leq l_1, |m_2| \leq l_2, |m| \leq l$ , and  $l$  equals one of  $(l_1 + l_2), (l_1 + l_2 - 1), \dots, |l_1 - l_2|$ . This is known from previous work in angular momentum theory.

[0032] The next step is to consider, based on the values of  $k_{l_1}$  and  $k_{l_2}^{(p)}$ , which values of the C-G coefficients will be necessary in evaluating the sums over  $(l_1, m_1, l_2, m_2)$ . Again, from work in angular momentum theory, the following is an expression for the C-G coefficients:

$$C(l_1 l_2 l; m_1 m_2 m) = \delta_{m, m_1 + m_2} \left( (2l+1) \frac{(l+l_1-l_2)!(l-l_1+l_2)!(l_1+l_2-l)!(l+m)!(l-m)!}{(l_1+l_2+l+1)!(l_1-m_1)!(l_1+m_1)!(l_2-m_2)!(l_2+m_2)!} \right)^{1/2} \cdot \sum_q (-1)^{q+l_2+m_2} \frac{(l_2+l+m_1-q)!(l_1-m_1+q)!}{q!(l-l_1+l_2-q)!(l+m-q)!(q+l_1-l_2-m)!}$$

where  $q$  are the integers satisfying

$$0 \vee (l_2 - l_1 + m) \leq q \leq (l+m) \wedge (l_2 - l_1 + l)$$

where  $A \vee B = \max(A, B)$  and  $C \wedge D = \min(C, D)$ . As usual, the convention is that  $0! = 1$ . Other useful facts about C-G coefficients are that  $C(l_1 0 l; m_1 0 m) = \delta_{l_1 l} \delta_{m_1 m}$ , and that  $C(l_1 l_2 l; 0 0 0) = 0$  unless  $(l_1 + l_2 + l)$  is even.

[0033] Consider the  $l=0$ ,  $m=0$  factor (of  $L_{00}$ ) in  $r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})$ , the d.c. response, so to speak. Since  $l=0, l_1=l_2$ , and as  $m=0$ ,  $m_2 = -m_1 = \bar{m}_1$ , one can either evaluate

$$C(l_1 l_2 0; m_1 m_2 0) = \delta_{l_1 l_2} \delta_{m_1 \bar{m}_2} (-1)^{l_1 - m_1} / (2l_1 + 1)^{1/2}$$

or perform the above integration over incident-light directions

$$\int d\hat{\mathbf{n}}_i Y_{00}(\hat{\mathbf{n}}_i) Y_{l_1 m_1}^*(\hat{\mathbf{n}}_i) Y_{l_2 m_2}^*(\hat{\mathbf{n}}_i) = (-1)^{m_1} (4\pi)^{-1/2} \delta_{l_1 l_2} \delta_{m_1 m_2}$$

to obtain for  $l=0, m=0$ :

$$r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})_{00} = u(\hat{\mathbf{z}} \cdot \hat{\mathbf{n}}) L_{00} (4\pi)^{1/2} \sum_{l_1=0} k_{l_1} k_{l_1}^{(p)} (2l_1 + 1)^{-1} \sum_{m_1=-l_1}^{l_1} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1}^*(\beta, \alpha)$$

Thus, unlike the cases of Lambertian and specular reflection, even for uniform lighting conditions, the reflected light is a function of the surface normal  $\hat{\mathbf{n}}$ . This is in addition to the effect of the nonlinearity due to the  $u(\hat{\mathbf{z}} \cdot \hat{\mathbf{n}})$  term, which enters in all cases. Do note, however, that  $k_{l_1}$  decreases as  $1/l_1^2$ , while  $k_{l_1}^{(p)}$  changes comparatively slowly until  $l_1 > p^{\frac{1}{2}}$ , where upon it falls roughly as  $\exp -l_1^2/p$  reaching  $2^{-p}$  by  $l_1 = p$ , and then falls off as  $1/l_1^{1+p}$  for  $l_1 > p$ . Thus, the major  $\hat{\mathbf{n}}$ -dependent term in the  $(l=0, m=0)$ -component of  $r_p$  will be the  $l_1=1$  term in the above sum. (The sum over  $m_1$  is, of course, real.) For the case of  $r_p', k_{l_1}^{(0)}$  falls off as  $1/l_1$ .

**[0034]** One might wonder how, if the lumination is uniform in solid angle ( $L_m=0, l>0$ ), one could possibly have an

angular dependence in the reflected light. However, any scattering which depends on, and only on, the angle to the surface normal will exhibit a non-uniform distribution when viewed from different directions. For example, if light were scattered only in the direction of the surface normal, regardless of incident angle, then a sphere would appear dark except for an intense bright emission from the center of the apparent disk. This is easily seen analytically. Choosing  $\hat{n}_e = \hat{z}$ , the expressions for  $r_s, r_L, r_p$  have the generic form

$$r(\hat{n}) = \int d\hat{n}_i L(\hat{n}) g(\hat{n}_i, \hat{n}) = \sum_{lm} L_{lm} G_{lm}(\hat{n})$$

$$g(\hat{n}_i, \hat{n}) = \sum_{lm} G_{lm}(\hat{n}) Y_{lm}^*(\hat{n}_i)$$

[0035] Thus  $G_{00} = \int d\hat{n}_i g(\hat{n}_i, \hat{n}) / (4\pi)^{1/2}$  and since  $g_p$  depends on  $(\hat{n}_i \cdot \hat{n})$ ,  $(\hat{n}_e \cdot \hat{n}_i)$ , and  $(\hat{n}_e \cdot \hat{n})$ , the integral over  $\hat{n}_i$ ,  $(4\pi)^{1/2} G_{00}$ , will be a function of  $(\hat{n} \cdot \hat{n}_e) = (\hat{n} \cdot \hat{z})$ . Hence, although  $G_{00}$  is independent of the azimuthal orientation of  $\hat{n}$ , in most cases it will depend on  $\hat{n}$ 's polar orientation. Since  $g_L$  depends only on  $(\hat{n}_i \cdot \hat{n})$ ,  $G_{00}$  for the Lambertian model is independent of  $\hat{n}$ ; the tight coupling between the incident and reflected light in specular reflection yields  $G_{00} = Y_{00}(\hat{n}_e) = (4\pi)^{-1/2}$  a constant. However, for most  $g(\hat{n}_i, \hat{n})$ ,

$G_{00} = G_{00}(\hat{\mathbf{n}} \cdot \hat{\mathbf{z}})$ . Without the  $(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}})$  constraint, this directional dependence can be suppressed; however, scattering from very oblique surfaces can be enhanced as much as a factor of 2.

[0036] See also the coefficients of  $L_{1m}$  and  $L_{2m}$  in the expression for  $r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})$ . For  $l=1$ , the only possible values of  $(l_1, l_2)$  are  $l_1 = (l_2 + 1)$  and  $l_2 = (l_1 + 1)$ ,  $(l_1 \wedge l_2)$  starting from zero:  $l_1 = l_2$  is out as  $l_1 + l_2 + 1 = 2l_1 + 1$ , which, being odd, renders  $C(l_1 l_1 l; 000) = 0$ ; and  $l_1 = l_2 + n$  or  $l_2 = l_1 + n$  yields a smallest  $l$  of  $n$ , hence  $n \geq 2$  are out for  $l = 1$ .

Also, as

$$C(l_1 l_2 l; m_1 m_2 m) = (-1)^{l_1 + l_2 - l} C(l_1 l_2 l; m_1 m_2 m)$$

(exchange), one can easily obtain the  $l_1 = (l_2 + 1)$  case. Using Rose's expression, find:

$$C(l_1(l_1 + 1)l; m_1(m - m_1)m) = (-1)^{l_1 - m_1} \left( \frac{6(2l_1)!(1 + m + l_1 - m_1)!(1 - m + l_1 + m_1)!}{(2l_1 + 3)!(l_1 - m_1)!(l_1 + m_1)!(1 + m)!(1 - m)!} \right)^{1/2}$$

in which,  $-l_1 \leq m_1 \leq l_1$  and  $-1 \leq m \leq 1$ . Using this, one can obtain for the  $(l = 1, m)$  term in the Phong-model reflectance:

$$\begin{aligned} r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})_{1m} &= u(\hat{\mathbf{z}} \cdot \hat{\mathbf{n}}) L_{1m} (-1)^m (4\pi/3)^{1/2} \sum_{l_1} \frac{6(l_1 + 1)}{(2l_1 + 3)(2l_1 + 2)(2l_1 + 1)} \\ &\cdot \sum_{m_1} (k_{l_1} k_{l_1+1}^{(p)} Y_{l_1+m_1}(\hat{\mathbf{n}}) Y_{l_1+1m_1-m}^*(\beta, \alpha) + k_{l_1+1} k_{l_1}^{(p)} Y_{l_1+1m_1-m}^*(\hat{\mathbf{n}}) Y_{l_1m_1}(\beta, \alpha)) \\ &\cdot \left( \frac{(1 + m + l_1 - m_1)!(1 - m + l_1 + m_1)!}{(l_1 - m_1)!(l_1 + m_1)!(1 + m)!(1 - m)!} \right)^{1/2} \end{aligned}$$

In this ( $l = 1$ ) expression, the sum on  $l_1$  runs from  $l_1 = 0$ , and the sum on  $m_1$  runs from  $m_1 = -l_1$  to  $m_1 = +l_1$ . As with  $l=0$ , the terms in the  $l_1$  sum fall off as  $1/l_1^{3+p}$  for  $l_1 > p$ .

[0037] For  $l = 2$ , the Clebsch-Gordon coefficients are more complicated. We will simply state the result for this component of the reflectance. Note that only  $l_2 = l_1 + 2$ ,  $l_1 = l_2 + 2$ , and  $l_1 = l_2$  must be considered, and that the first two are, of course, closely related. The third case is more involved and requires separate expressions for  $m = \pm 2$ ,  $m = \pm 1$ , and  $m=0$ .

$$\begin{aligned}
 r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})_{2m} = & u(\hat{\mathbf{z}} \cdot \hat{\mathbf{n}}) L_{2m} (-1)^m (4\pi/5)^{1/2} \left( \sum_{l_1} \frac{60(2l_1)!(l_1+2)!}{(2l_1+5)!l_1!} \right. \\
 & \cdot \sum_{m_1} (k_1 k_{l_1+2}^{(p)} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1+2m_1-m}^*(\beta, \alpha) + k_{l_1+2} k_l^{(p)} Y_{l_1+2m_1-m}^*(\hat{\mathbf{n}}) Y_{l_1 m_1}(\beta, \alpha)) \\
 & \cdot \left( \frac{(2+m+l_1-m_1)!(2-m+l_1+m_1)!}{(2+m)!(2-m)!(l_1+m_1)!(l_1-m_1)!} \right)^{1/2} \\
 & + \sum_{l_1} k_l k_l^{(p)} \frac{30(2l_1-2)!}{(2l_1+3)!} (((l_1-1)l_1(l_1+1)(l_1+2)))^{1/2} \\
 & \cdot \sum_{m_1} ((l_1+m_1)(l_1+m_1-1)(l_1-m_1+2)(l_1-m_1+1))^{1/2} \\
 & \cdot (\delta_{m,2} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1-2}^*(\beta, \alpha) + \delta_{m,-2} Y_{l_1 m_1}^*(\hat{\mathbf{n}}) Y_{l_1 m_1-2}(\beta, \alpha)) \\
 & + (l_1(l_1+1))^{1/2} \sum_{m_1} ((l_1+m_1)(l_1-m_1+1))^{1/2} (1-2m_1) \\
 & \cdot (\delta_{m,1} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1-1}^*(\beta, \alpha) - \delta_{m,-1} Y_{l_1 m_1}^*(\hat{\mathbf{n}}) Y_{l_1 m_1-1}(\beta, \alpha)) \\
 & + \frac{2}{3} l_1(l_1+1) \sum_{m_1} (l_1(l_1+1)-3m_1^2) \delta_{m,0} Y_{l_1 m_1}(\hat{\mathbf{n}}) Y_{l_1 m_1}^*(\beta, \alpha) \Big)^{1/2}
 \end{aligned}$$

[0038] In this ( $l=2$ ) expression, the terms in the first ( $l_2 = l_1+2$ ), second ( $l_2 = l_1$ ,  $m = 2$ ), and fourth ( $l_2 = l_1$ ,  $m = 0$ )  $l_1$ -summations all fall off as  $1/l_1^{3+p}$ , as for  $l=0,1$ , while

[illegible]

was useful in the above derivation.

[0039] From the foregoing, the function of broadened specular reflectance,  $r_p(\hat{\mathbf{z}}, \hat{\mathbf{n}})$ , can be described in terms of a summation of the spherical harmonic components of the light incident upon the model.

-24-



the components creates a rendered image that most nearly approximates the input image.

**[0041]** The method of deriving the reflectance function described above is used in the object recognition process. To match an input image with an object model in the database, the object model must be positioned to match the position of the input image. There are standard methods of performing this function known in the art, though a preferred method is disclosed in U.S. Patent Application Serial Number 09/538,204 (Attorney Docket No. NECI 1083, 13414), filed 30 March 2000, entitled "Method for Matching a Two Dimensional Image to One of a Plurality of Three Dimensional Candidate Models in a Database", the disclosure of which is hereby incorporated by reference.

**[0042]** Once positioned, images of the model are rendered under each of one or more spherical harmonic components of light as described above. These component rendered images can be linearly combined to arrive at an optimal rendered image of the model. An optimal rendered image means a rendered image that most closely approximates the input image. The comparison of the various rendered images to the input image is made by calculating the Euclidian Distance between the images, in a manner that is known in

the art. The optimization is performed by a least squares fit technique between the vector that is the reflectance function and the matrix that represents the input image, also known in the art.

**[0043]** Other steps follow the optimization in the recognition process. One is computing some measure of similarity between the input image and the optimal rendered image for each model. This is accomplished by measuring the Euclidian Distance between each optimal image and the input image. The object model whose optimal rendered image is nearest the input image, assuming some threshold value of similarity must be reached, is a match to the input image.

**[0044]** While the reflectance function of the present invention is described in application to the field of machine vision, it is not limited to that field. There are numerous parallels between this work and advances in the field of computer graphics, and our invention has applicability there as well. The field of computer graphics is continually striving to produce more realistic rendered images based on object models that exist only in the computer. The reflectance function of the present invention advances that work as well.

[0045] Our invention has been described herein with reference to a particular exemplary embodiment. Certain alterations and modifications may be apparent to those skilled in the art, without departing from the scope of the invention. The exemplary embodiment is not meant to be limiting on the scope of the invention, which is defined by the appended claims.